
CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL

2016 Trial Examination

FORM VI

MATHEMATICS 2 UNIT

Tuesday 9th August 2016

General Instructions

- Reading time — 5 minutes
- Writing time — 3 hours
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 100 Marks

- All questions may be attempted.

Section I – 10 Marks

- Questions 1–10 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II – 90 Marks

- Questions 11–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your candidate number on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Eleven.

Checklist

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Reference sheet
- Candidature — 88 boys

Examiner

PKH

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

What are the solutions of $x^2 - 3x + 1 = 0$?

(A) $x = \dfrac{3 \pm \sqrt{5}}{2}$

(B) $x = \dfrac{-3 \pm \sqrt{13}}{2}$

(C) $x = \dfrac{3 \pm \sqrt{13}}{2}$

(D) $x = \dfrac{-3 \pm \sqrt{5}}{2}$

QUESTION TWO

What is the limiting sum for the infinite geometric series $12 - 6 + 3 - \ldots$?

(A) 24

(B) 8

(C) -8

(D) -12

QUESTION THREE

What is the derivative of $\dfrac{2}{x}$?

(A) $2 \ln x$

(B) $\ln 2x$

(C) $-\dfrac{2}{x^2}$

(D) $\dfrac{2}{x^2}$

Examination continues next page ...

QUESTION FOUR

Which of the following is a primitive of e^{2x}?

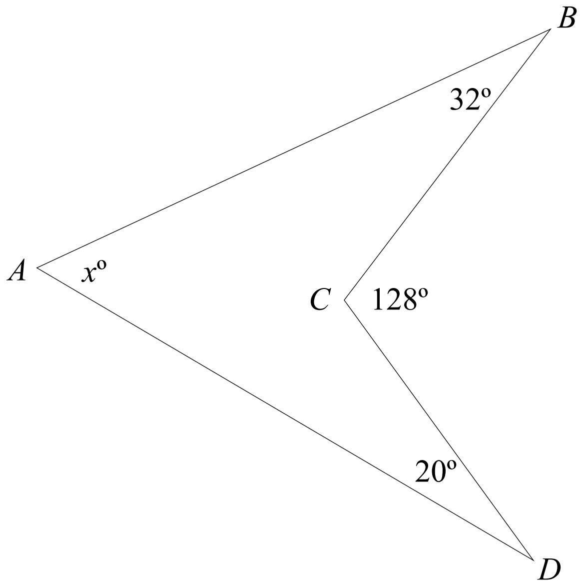
(A) $(2x+1)e^{2x+1}$

(B) $2e^{2x}$

(C) $\dfrac{e^{2x+1}}{2x+1}$

(D) $\dfrac{e^{2x}}{2}$

QUESTION FIVE



What is the value of x in the diagram above?

(A) 66

(B) 76

(C) 64

(D) 86

QUESTION SIX

Simplify $\log_4 54 - 2\log_4 3$.

(A) $\log_4 9$

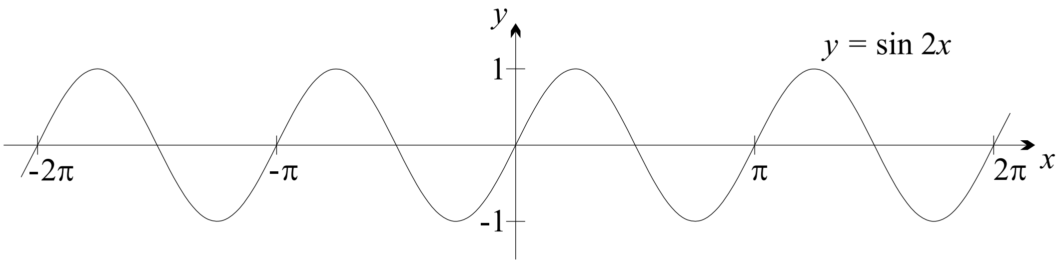
(B) $\log_4 48$

(C) $\log_4 6$

(D) 1

Examination continues overleaf ...

QUESTION SEVEN



The graph of $y = \sin 2x$ is drawn. How many solutions does the equation $\frac{1}{6}x = \sin 2x$ have?

(A) 3

(B) 4

(C) 7

(D) 8

QUESTION EIGHT

Consider the points $A(1, -2)$ and $B(3, 6)$. What is the equation of the perpendicular bisector of AB?

(A) $y - 2 = -\frac{1}{4}(x - 2)$

(B) $y - 2 = 4(x - 2)$

(C) $y - 4 = -1(x - 1)$

(D) $y + 2 = -\frac{1}{4}(x - 1)$

QUESTION NINE

What is the greatest value of $\dfrac{20}{4\sin^2\theta + 2\cos^2\theta}$ for $0 \le \theta \le \dfrac{\pi}{2}$?

(A) 10

(B) 5

(C) 20

(D) $\dfrac{20}{6}$

QUESTION TEN

Which of the following is a correct simplification of $\dfrac{\cos(\pi - x)}{\cos\left(\dfrac{\pi}{2} - x\right)}$?

(A) $\cos\dfrac{\pi}{2}x$

(B) $-\tan x$

(C) $-\cot x$

(D) $\tan x$

End of Section I

Examination continues overleaf ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION ELEVEN (15 marks) Use a separate writing booklet. **Marks**

(a) Calculate $3e^{1\cdot 5}$ correct to 3 decimal places. **1**

(b) Find the gradient of the line $3y - 2x = 6$. **1**

(c) Factorise $9a^2 - 16$. **1**

(d) Differentiate $x^3 e^x$. **2**

(e) Differentiate $(3 + \sin x)^4$. **2**

(f) Solve the inequation $5 - 2x \geq 14$. **2**

(g) Solve $|2x - 5| = 7$. **2**

(h) Find the coordinates of the focus of the parabola $(x - 2)^2 = 8y + 16$. **2**

(i) Solve $2\sin\theta = -1$ for $0 \leq \theta \leq 2\pi$. **2**

QUESTION TWELVE (15 marks) Use a separate writing booklet. **Marks**

(a) Make y the subject of the equation $x = \log_3 y$. **1**

(b) Find $\displaystyle\int \frac{4x^3}{2 + x^4}\,dx$. **1**

(c) Differentiate $\dfrac{x}{\sin x}$. **2**

(d) Evaluate $11 + 16 + 21 + \cdots + 101$. **3**

(e) The quadrilateral $ABCD$ has vertices $A(0, 4), B(4, 8), C(-1, -4)$ and $D(-5, -8)$.

(i) Show that $ABCD$ is a parallelogram. **2**

(ii) Find the equation of line BC, leaving your answer in the form $ax + by + c = 0$. **2**

(iii) Find the perpendicular distance from A to line BC. **2**

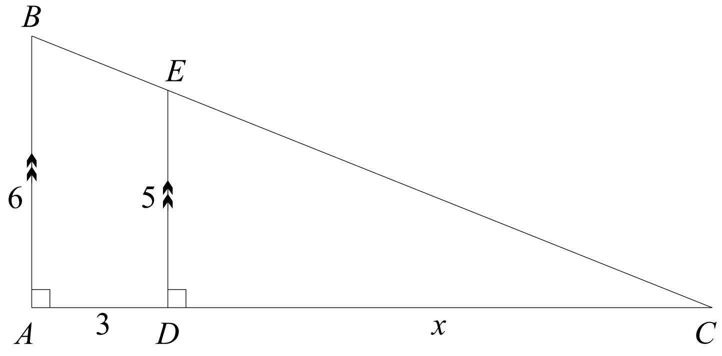
(iv) Find distance BC. **1**

(v) Hence find the area of $ABCD$. **1**

Examination continues next page . . .

QUESTION THIRTEEN (15 marks) Use a separate writing booklet. **Marks**

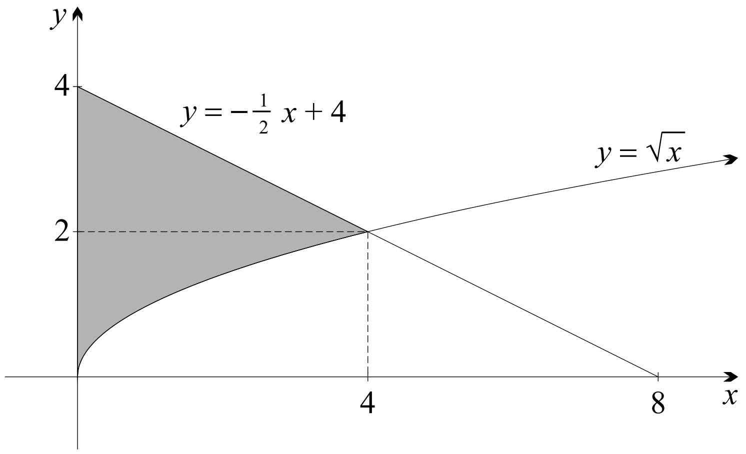
(a)



(i) Prove that $\triangle ABC \parallel \triangle DEC$ in the diagram above. **1**

(ii) Find the value of x, giving reasons. **2**

(b)



Find the shaded area in the diagram above. **3**

(c) A person walks on the true bearing of 050° for 20km from point P and stops at point A. Another person walks for 30km on a bearing of 110° from point P and stops at point B.

(i) Represent this information on a neat diagram. **1**

(ii) Find the distance AB to the nearest kilometre. **2**

(iii) Find the bearing of A from B to the nearest degree. **2**

(d) The volume V is the number of litres of water in a tank at time t minutes. Water is flowing into the tank at a rate given by $\dfrac{dV}{dt} = \dfrac{4}{2t+1}$ litres per minute. At time $t = 0$ the water begins to flow into an empty tank. How much water is in the tank after 5 minutes, to the nearest tenth of a litre? **2**

(e) Use the trapezoidal rule with 3 function values to estimate $\displaystyle\int_1^3 2^x \, dx$. **2**

Examination continues overleaf ...

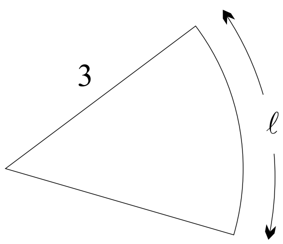
QUESTION FOURTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Differentiate $\log_e(e^x + 2)$. **2**

(b) A sum of \$20 000 is invested at a fixed rate of interest, compounded annually. After 5 years the principal has grown to \$28 567. **3**

Find the annual rate of interest to the nearest tenth of one percent.

(c) **2**



The sector, shown in the diagram above, has an area of 36 square units and a radius of 3 units. Find the arc length ℓ.

(d) Solve the equation $\tan^2\theta + \sqrt{3}\tan\theta = 0$ for $0 \le \theta \le 2\pi$. **2**

(e) A particle is moving in a straight line with velocity given by $\dot{x} = 3t^2 - 9t$ where t is measured in seconds and x is measured in metres. Its displacement from the origin is initially 10 metres.

(i) Find the displacement x as a function of t. **2**

(ii) Find the displacement when the acceleration is zero. **2**

(iii) Find the average speed during the first 4 seconds. **2**

QUESTION FIFTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Find the volume formed when $y = \sec 2x$ is rotated about the x-axis from $x = 0$ to $x = \dfrac{\pi}{8}$. **2**

(b) Find $\displaystyle\int \left(\sqrt[3]{x-9}\right)^2 \, dx$. **2**

(c) The population P of a town is growing at a rate proportional to its size at any time, so that $\dfrac{dP}{dt} = kP$, for some constant k. At the beginning of 2010 the town's population was 23 000 and at the beginning of 2016 its population had grown to 28 000.

(i) Show that $P = Ae^{kt}$ satisfies the equation $\dfrac{dP}{dt} = kP$. **1**

(ii) Find the value of A. **1**

(iii) Find the value of k. **2**

(iv) Estimate, to the nearest hundred, what the population will be at the beginning of 2025. **1**

(v) During which year will the population be double the size it was at the beginning of 2010? **2**

(d) A person borrows \$400 000 and makes regular monthly repayments of $\$M$. The interest rate is 6% per annum compounded monthly. The loan is taken over a period of 20 years. Let A_n be the amount owing after n months, just after a repayment has been made.

(i) Find an expression for A_2. **1**

(ii) Find the monthly payment M to the nearest cent. **3**

Examination continues overleaf ...

QUESTION SIXTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) Consider the function $y = x^5 - 80x$.

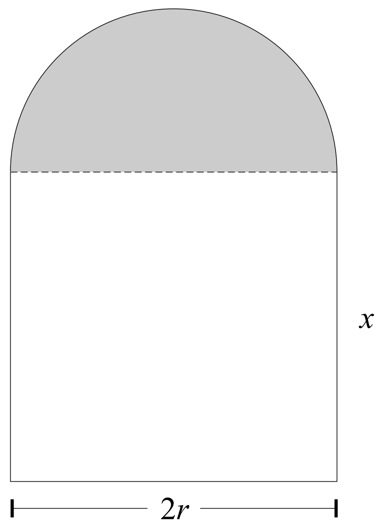
(i) Find the x-intercepts. **1**

(ii) Find the stationary points and determine their nature. **2**

(iii) Find the point of inflexion. **2**

(iv) Draw a neat sketch of the function, showing the above information. **2**

(b)



A large window is constructed in the shape of a rectangle with a semicircle on top, as in the diagram above. The glass forming the semicircle is opaque and the glass forming the rectangle is clear. The height of the rectangle is x metres and the radius of the semicircle is r metres. The perimeter of the entire window is 12 metres.

(i) Show that $x = 6 - \frac{\pi}{2}r - r$. **2**

(ii) The window is constructed so that the area of the rectangle, made of clear glass, is maximised. **3**
Show that $r = \dfrac{6}{\pi + 2}$.

(c) The cubic function $y = ax^3 + bx^2 + cx + d$ has two stationary points and one point of inflexion. **3**

Prove that the x-coordinate of the point of inflexion is located at the average of the x-coordinates of the two stationary points.

End of Section II

END OF EXAMINATION

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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

Question Eight

A ◯ B ◯ C ◯ D ◯

Question Nine

A ◯ B ◯ C ◯ D ◯

Question Ten

A ◯ B ◯ C ◯ D ◯

SOLUTION TO UNIT TRIAL SGS 2016

Question 1

$x^2 - 3x + 1 = 0$

$x = \dfrac{3 \pm \sqrt{9-4}}{2}$

$x = \dfrac{3 \pm \sqrt{5}}{2}$ (A)

Question 2

$S_\infty = \dfrac{a}{1-r}$

$S_\infty = \dfrac{12}{1 - -\frac{1}{2}}$

$= 12 \times \frac{2}{3}$

$= 8$ (B)

Question 3

$y = 2x^{-1}$

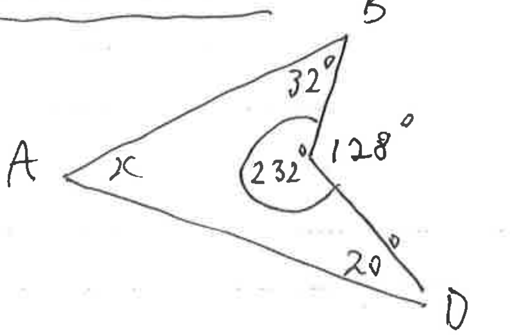
$y' = -2x^{-2}$

$= -\dfrac{2}{x^2}$ (C)

Question 4

$\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$ (D)

Question 5



$32 + 20 + 232 + x = 360°$

$286 + x = 360$

$x = 76$ (B)

6 $\log_4 54 - 2\log_4 3$

$= \log_4 54 - \log_4 9$

$= \log_4 6$ (C)

7. The line $y = \frac{1}{6}x$ when drawn carefully cuts the curve 7 times. (C)

8 $m = \dfrac{6 - -2}{3-1} = 4$

$m_\perp = -\frac{1}{4}$ $M = (2, 2)$

Eqn is

$y + 2 = -\frac{1}{4}(x - \ )$ (A)

9. $4\sin^2\theta + 2\cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta + 2\sin^2\theta$

$= 2 + 2\sin^2\theta$

Least when $\sin\theta = 0$

Max of expression is $\frac{20}{2} = 10$

(A)

10 $\dfrac{\cos(\pi - x)}{\cos(\frac{\pi}{2} - x)}$

$= \dfrac{-\cos x}{\sin x}$ (C)

$= -\cot x$

QUESTION ELEVEN

(a) $3e^{1.5}$ ✓

$= 13.445$ (3 dp)

(b) $3y - 2x = 6$

$y = \frac{2}{3}x + 2$

Gradient is $\frac{2}{3}$ ✓

(c) $9a^2 - 16$

$= (3a-4)(3a+4)$ ✓

(d) $y = \underset{u}{x^3}\,\underset{v}{e^x}$

$y' = uv' + vu'$

$= x^3e^x + 3x^2e^x$ ✓✓

$= x^2e^x(x+3)$

(e) $y = (3+\sin x)^4$

$y' = 4(3+\sin x)^3\cos x$ ✓ ✓

(f) $5 - 2x \geqslant 14$

$-2x \geqslant 9$ ✓

$x \leqslant -4.5$ ✓

(g) $|2x-5| = 7$

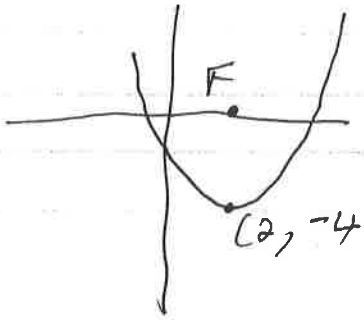
$2x-5 = 7$ OR $2x-5 = -7$

$x = 6$ OR $x = -1$

✓ ✓

(h) $(x-2)^2 = 4(y+4)$

$a = 4$, $V = (2,-4)$ ✓



$F = (2, 0)$ ✓

(i) $2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$

$\theta = \pi + \frac{\pi}{6}$ or $2\pi - \frac{\pi}{6}$

$= \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

✓ ✓

Question 12

3

(a) $x = \log_3 y$

$y = 3^x$ ✓

(b) $\int \frac{4x^3}{2+x^4}\,dx$ ✓

$= \ln(2+x^4) + C$

(c) $y = \frac{x}{\sin x}$ $\quad - u$, $- v$

$y' = \frac{vu' - uv'}{v^2}$ ✓

$= \frac{\sin x \cdot 1 - x \cos x}{\sin^2 x}$ ✓

(d) $T_n = 101$

$a + (n-1)d = 101$

$11 + (n-1)5 = 101$ ✓

$11 + 5n - 5 = 101$

$5n = 95$

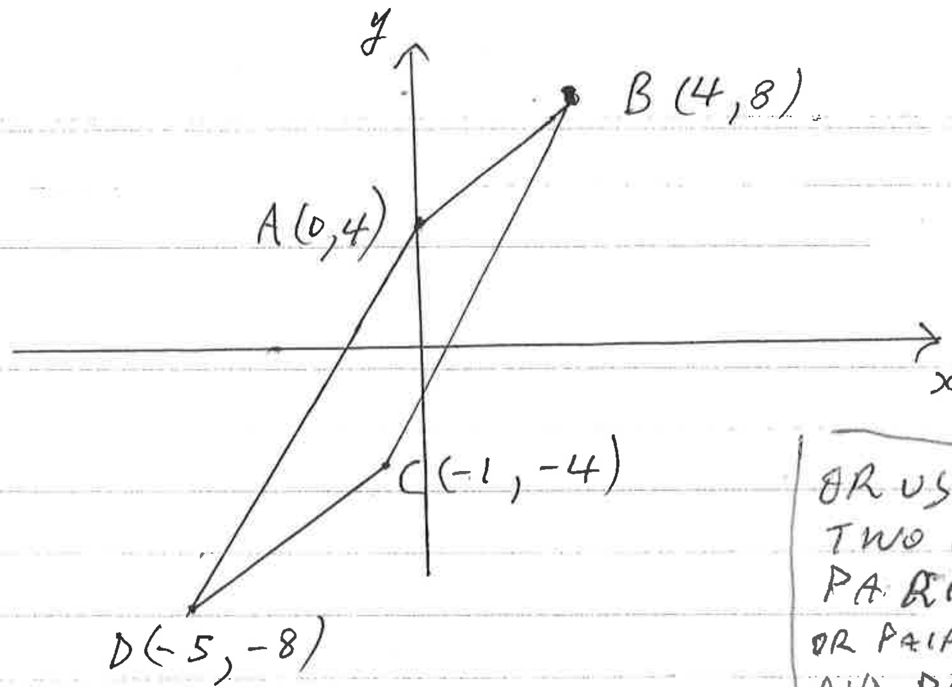
$n = 19$ ✓

$S_n = \frac{n}{2}(a+l)$

$S_{19} = \frac{19}{2}(11+101)$

$= 1064$ ✓

(e)



4

OR USE
TWO PAIRS OF
PARALLEL sides
OR PAIR OF EQUAL
AND PARALLEL sides

(i) Let M = mid point of AC.

$$M = \left(\frac{0+-1}{2}, \frac{4+-4}{2}\right) = \left(-\frac{1}{2}, 0\right)$$ ✓

Mid point of $BD = \left(-\frac{1}{2}, 0\right)$

So diagonals bisect each other ✓

So $ABCD$ is a parallelogram

(ii) $$m(BC) = \frac{8--4}{4--1} = \frac{12}{5}$$

Eqn of line BC is

$$y+4 = \frac{12}{5}(x+1)$$ ✓

$$5y + 20 = 12x + 12$$ ✓

$$12x - 5y - 8 = 0$$ ✓

(iii) $$d_\perp = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$$

$$= \frac{|12\times 0 - 5\times 4 - 8|}{\sqrt{12^2 + (-5)^2}} = \frac{28}{13}$$ ✓

(iv) $$d(BC) = \sqrt{(4-(-1))^2 + (8-(-4))^2}$$

$$= 13$$ ✓

(v) Area $= b \times h$

$$= 13 \times \frac{28}{13} = 28.$$ ✓

Question 13 (5)

a (i) $\angle ACB$ is common

$\angle BAC = \angle EDC = 90°$ ✓

$\triangle ABC \parallel\!\!| \triangle DEC$ (AAA) Must have this.

(ii) $\frac{x}{x+3} = \frac{5}{6}$ ({corresponding / matching} sides in similar $\triangle$'s) ✓

$6x = 5x + 15$

$x = 15$ ✓

(b) $A = \int_a^b y_1 - y_2 \, dx$ ✓

$= \int_0^4 -\frac{1}{2}x + 4 - x^{\frac{1}{2}} \, dx$

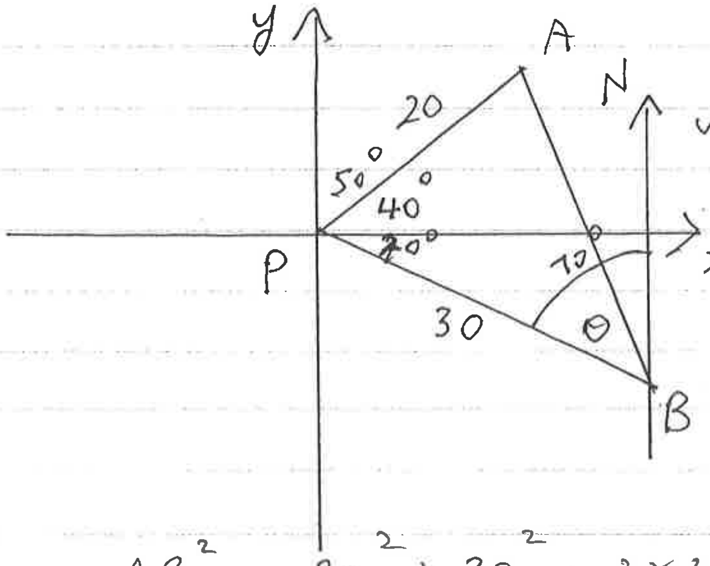
$= \left[-\frac{x^2}{4} + 4x - \frac{2}{3} x^{\frac{3}{2}} \right]_0^4$ ✓

$= -\frac{16}{4} + 16 - \frac{2}{3} \times 8 - [0]$

$= \frac{20}{3} u^2$ ✓

(c)

(i)



✓

$\angle APB = 60°$

$\cos 60° = \frac{1}{2}$

(ii) $AB^2 = 20^2 + 30^2 - 2 \times 20 \times 30 \cos 60°$ ✓

$AB^2 = 1300 - 600$ ✓

$AB = \sqrt{700} \doteqdot 26.4575$

$= 26$ km (to nearest km)

6

(iii) $\cos\theta = \dfrac{30^2 + (26.4575)^2 - 20^2}{2 \times 30 \times 26.4575}$ ✓

$\theta = 40.89 = 41^\circ$ to nearest degree

Bearing $= 360^\circ - 70 + \theta$

$\approx 331^\circ$ (nearest degree) ✓

(iv) $\dfrac{dV}{dt} = \dfrac{4}{2t+1}$

$V = 4\,\dfrac{\ln(2t+1)}{2} + C$

$V = 2\ln(2t+1) + C$ ✓

When $\left.\begin{array}{l} t=0 \\ V=0 \end{array}\right\}$ $0 = 2\ln 1 + C$

$C = 0$

[Must show calculation of C]

$V = 2\ln(2t+1)$

When $t = 5$ $V = 2\ln 11$

$= 4.795\ldots$ ✓

$= 4.8$ L (to nearest tenth of a litre)

(v)

x	1	2	3
y	2	4	8

✓

$\int_1^3 2^x\,dx = \dfrac{2-1}{2}(2+4) + \dfrac{3-2}{2}(4+8)$

$= 9$ ✓

Question 14

(a) $y = \ln(e^x + 2)$

$$y' = \frac{e^x}{e^x + 2}$$

(b) $P = A\left(1+\frac{r}{100}\right)^n$

$$28567 = 2000\left(1+\frac{r}{100}\right)^5$$

$$\left(1+\frac{r}{100}\right)^5 = \frac{28567}{20000}$$

$$1+\frac{r}{100} = \sqrt[5]{\frac{28567}{20000}}$$

$$1+\frac{r}{100} = 1.0739\ldots$$

$$\frac{r}{100} = 0.0739\ldots$$

$$r = 7.39\ldots$$

So rate is 7.4%

(c) $A = \frac{1}{2}r^2\theta = 36$

$$\frac{1}{2} \times 9 \times \theta = 36$$

$$\theta = 8$$

$$l = r\theta$$
$$= 3 \times 8$$
$$= 24 \text{ units}$$

(d) $\tan^2\theta + \sqrt{3}\tan\theta = 0$ for $0 \le \theta \le 2\pi$

$$\tan\theta(\tan\theta + \sqrt{3}) = 0$$

$\tan\theta = 0$ or $\tan\theta = -\sqrt{3}$

$\theta = 0, \pi, 2\pi$ or $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

8

(e) (i) $\dot{x} = 3t^2 - 9t$

$$x = t^3 - \frac{9}{2}t^2 + C$$

$t=0$, $x=10$: $\quad 10 = 0 - 0 + C$

$$C = 10$$

$$x = t^3 - \frac{9}{2}t^2 + 10$$

(ii) $\ddot{x} = 6t - 9 \qquad \ddot{x} = 0 \qquad t = \frac{3}{2}$

When $t = \frac{3}{2}$ $\quad x = \left(\frac{3}{2}\right)^3 - \frac{9}{2} \times \left(\frac{3}{2}\right)^2 + 10$

$$x = \frac{27}{8} - \frac{81}{8} + \frac{80}{8}$$

$$x = \frac{26}{8} = \frac{13}{4}$$

(iii) The particle can change direction when $\dot{x} = 0$

$$3t^2 - 9t = 0$$

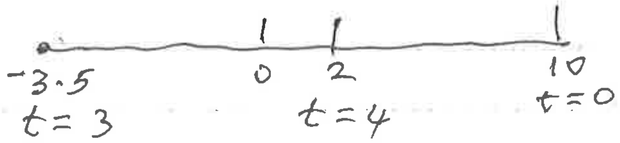
$$3t(t-3) = 0$$

$$t = 0 \text{ or } t = 3$$

$t = 0$, $\quad x = 10$

$t = 3$, $\quad x = 27 - \frac{9}{2} \times 9 + 10 = -3.5$

$t = 4$, $\quad x = 64 - \frac{9}{2} \times 16 + 10 = 2$



Total distance travelled $= 13.5 + 5.5 = 19$

Average speed over first 4 secs $= \frac{19}{4} = 4.75$ m/s

Question 15

(a) $V = \pi \int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$

$= \pi \left[\frac{\tan 2x}{2} \right]_0^{\frac{\pi}{8}}$ ✓

$= \frac{\pi}{2} \left[\tan\frac{\pi}{4} - \tan 0 \right]$

$= \frac{\pi}{2}$ unit^2 ✓

(b) $\int \left(\sqrt[3]{x-9} \right)^2 dx$

$= \int (x-9)^{\frac{2}{3}} dx$ ✓

$= \dfrac{(x-9)^{\frac{5}{3}}}{\frac{5}{3}} + C$

$= \frac{3}{5}(x-9)^{\frac{5}{3}} + C$ ✓

(c) (i) $P = Ae^{kt}$

$\frac{dP}{dt} = kAe^{kt}$

$\frac{dP}{dt} = kP$ ✓

(ii) $P = Ae^{kt}$

In 2010, $t=0$ $\quad 23000 = Ae^0$

$P = 23000$ $\quad A = 23000$ ✓

(iii) In 2016 $t=6$ $\quad 28000 = 23000e^{6k}$ ✓

$P = 28000$ $\quad k = \frac{1}{6}\ln\left(\frac{28}{23}\right)$ ✓

$k \doteqdot 0.032785\ldots$

(iv) $P = 23000\,e^{15K}$

$P \doteq 37600$ (to nearest hundred) ✓

(V) $P = 23000\,e^{Kt}$

$t = ?$

$P = 46000$ (doubled)

$46000 = 23000e^{Kt}$ ✓

$e^{Kt} = 2$

$Kt = \ln 2$

$t = \frac{1}{K}\ln 2$

$t = 21\cdot14\ldots$ ✓

Doubles during the 22nd year ie 2031.

(c) (i) $A_1 = 400{,}000 \times 1{\cdot}005 - M$ 6% pa $= 0\cdot005$ monthly

$A_2 = (400{,}000 \times 1{\cdot}005 - M)\,1{\cdot}005 - M$ ✓

* Must show 3 terms.

$A_2 = 400{,}000 \times 1{\cdot}005^2 - M\,(1 + 1{\cdot}005)$ *

(ii) $A_n = 400{,}000 \times 1{\cdot}005^n - M\,(1 + 1{\cdot}005 + 1{\cdot}005^2 + \cdots + (1{\cdot}005)^{n-1})$

$A_{240} = 400\,000 \times (1{\cdot}005)^{240} - M(1 + 1{\cdot}005 + \cdots + 1{\cdot}005^{239})$ ✓

But $A_{240} = 0$

So $400{,}000 \times 1{\cdot}005^{240} = M\dfrac{a(r^n - 1)}{r-1}$ ✓

$400{,}000 \times 1{\cdot}005^{240} = M \times 1\dfrac{(1{\cdot}005^{240} - 1)}{0{\cdot}005}$

$M = \$2865{\cdot}72$ ✓

Question 16

(a) $y = x^5 - 80x$

(i) x intercepts where $y = 0$

$$x^5 - 80x = 0$$
$$x(x^4 - 80) = 0$$
$$x = 0, \sqrt[4]{80}, -\sqrt[4]{80}$$ ✓

(ii) $y' = 5x^4 - 80$

Stat pts where $y' = 0$

$$5x^4 = 80$$
$$x^4 = 16$$
$$x = \pm 2.$$ ✓

$$y'' = 20x^3$$

When $x = 2$, $y'' = 160 > 0$
Min pt at $(2, -128)$
When $x = -2$, $y'' = -160 < 0$
Max pt at $(-2, 128)$ ✓

(iii) Possible point of inflexion where $y'' = 0$
Table of values for y'' ie $x = 0$

x	-1	0	1
y''	-20	0	20

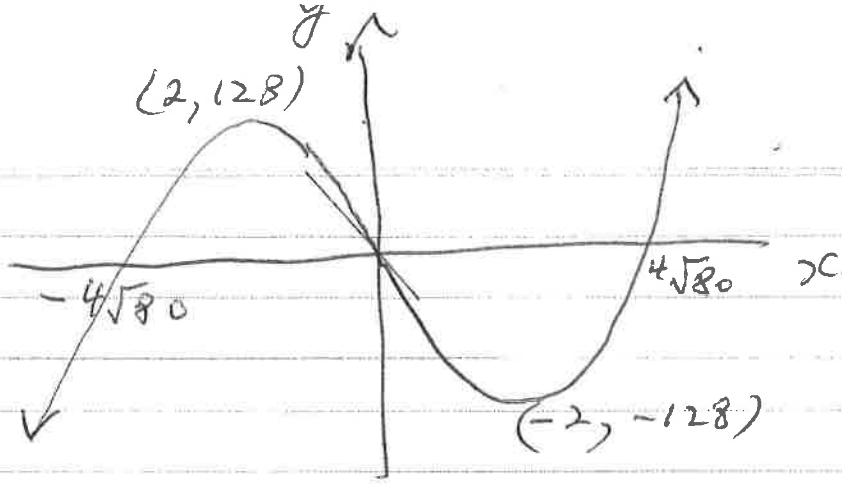
✓

There is a change in concavity at $x = 0$ ✓

So $(0, 0)$ is a point of inflexion

12

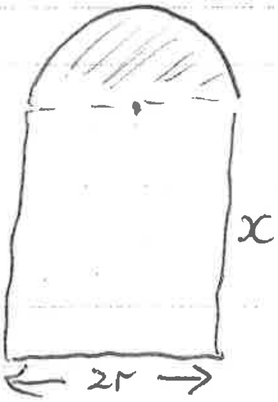
(iv)



✓ ✓

(b)

(i)



$P = 2r + \pi r + 2x$ ✓

$12 = 2r + \pi r + 2x$ ✓

$x = 6 - \frac{\pi}{2} r - r$

(ii)

$A = 2r\ x$

$= 2r\left(6 - \frac{\pi}{2} r - r\right)$

$A = 12r - \pi r^2 - 2r^2$

$A' = 12 - 2\pi r - 4r$ ✓

$A'' = -2\pi - 4 \quad < 0$ ✓

Max value where $A' = 0$

$$\left.\begin{aligned} 12 - 2\pi r - 4r &= 0 \\ 6 - \pi r - 2r &= 0 \\ 6 &= \pi r + 2r \end{aligned}\right\} ✓$$

$r = \frac{6}{\pi + 2}$

(c) $y = ax^3 + bx^2 + cx + d$

$y' = 3ax^2 + 2bx + c$ ✓

Let x co-ords of the stationary pts be α and β

α and β are roots of

$3ax^2 + 2bx + c = 0$

$\alpha + \beta = \Sigma$ roots

$\alpha + \beta = \frac{-2b}{3a}$

Average of α and β = $\frac{\alpha+\beta}{2} = \frac{-b}{3a}$ ✓

We are told that there is a point of inflexion.

This occurs when $y'' = 0$

$6ax + 2b = 0$

$x = \frac{-b}{3a}$ ✓

which is the average of α and β.